

Figure This!

Math Challenges for Families

How l o n g before you
b u r n?



Figure This! Helix can usually stay in the sun 8 minutes before being sunburned. Using a sun tan lotion product with SPF (Sun Protection Factor) 10 means that he can stay in the sun 8×10 , or 80 minutes before being burned. Helix put SPF 10 lotion on five minutes after he got to the beach. An hour later, he is burned. Why?

Hint: How much burning time was gone before the lotion was used?

Citizens need to understand what numbers represent in situations in their daily lives. Understanding such numerical values as SPF, the category of a hurricane, and electrical wiring codes is important for making informed decisions.

Since Helix was exposed to the sun for 5 minutes without protection, he had only 3 minutes left before he would burn. A sunscreen with an SPF of 10 would extend the time left to 30 minutes.

Answer:

Figure This!

Get Started:

How long was Helix at the beach before applying the sunscreen? How many more minutes of exposure could his skin withstand before burning with sunscreen? with no sunscreen?

Complete Solution:

An SPF 10 product used before going in the sun, and when the user normally burns in 8 minutes, provides about 8×10 , or 80 minutes of protection. Putting the lotion on 5 minutes after being in the sun uses $\frac{5}{8}$ of the time before being burned. Thus, $\frac{3}{8}$ of the normal protection time before burning is left. Because $(\frac{3}{8})(80)$ is 30, a person has 30 minutes before burning.

Note: The limits of safe exposure to sunlight vary by individual, season, and geographic location.

Try This:

- Do some research on the web to see the effects of extended exposure to sunlight.
- Determine what type of sun protection you should use.
- Find out what ultraviolet (UV) rays are and how they are measured.
- Consult a doctor, nurse, or pharmacist about potential allergies to certain sunscreens.

Additional Challenges:

(Answers located in back of booklet)

1. The chart below shows skin color and recommended SPF to avoid sunburn. Suppose there was another complexion called “extremely fair,” what SPF would be recommended for a person with that complexion?

Skin Color / Complexion	Recommended SPF to Avoid Sunburn
Dark	2
Medium	6
Light	15
Very Fair	30

2. A microwave oven can bake a potato in 5 minutes. To crisp the skin, a potato can be transferred to a regular oven set at 350° after 3 minutes in the microwave. If it normally takes 1 hour for a potato to bake in the regular oven, when will the potato be done?
3. Water flows into a pool spa at the rate of 14 gallons per minute. If the pool is one-third full and it holds 360 gallons, how long does it take to complete filling?

Things to Think About:

- How long can you usually stay in the sun before getting sunburned?
- What type of sun protection do you use? Why should you re-apply suntan lotion during the course of the day?
- In which parts of the world are severe sunburns most likely?
- Can you sunburn on a cloudy day?
- Would an SPF of 200 make any sense?
- Why do people sunburn faster when they are playing on the water, on snow, or at the beach?

Did You Know That?

- A tan is your skin’s reaction to the damage done by ultraviolet radiation.
- Infrared radiation from the sun also can be harmful to the skin, increasing the risk of skin cancer and causing premature aging.
- In North America, the peak hours of exposure to ultraviolet radiation are between 10:00 A.M. and 3:00 P.M.
- The exposure to ultraviolet rays increases at higher altitudes and in regions where the ozone layer is thinner.
- Many medicines, including certain antibiotics, can increase a person’s sensitivity to sunlight.
- The chemicals in some perfumes also can heighten sensitivity to sunlight.
- Symptoms of sunburn—such as reddened skin—can take up to 24 hours to appear.

Resources:

Magazine:

- Crabtree, K. “The Great Cover-up.” *Ski* (February 2001): 94–96.

Websites:

- www.htropic.com/sunfacts/index.shtml
- www.emedicine.com/EMERG/topic798.htm
- www3.utsouthwestern.edu/library/consumer/sunbm.htm



Figure This!
Math Challenges for Families

Why are airport runways numbered?

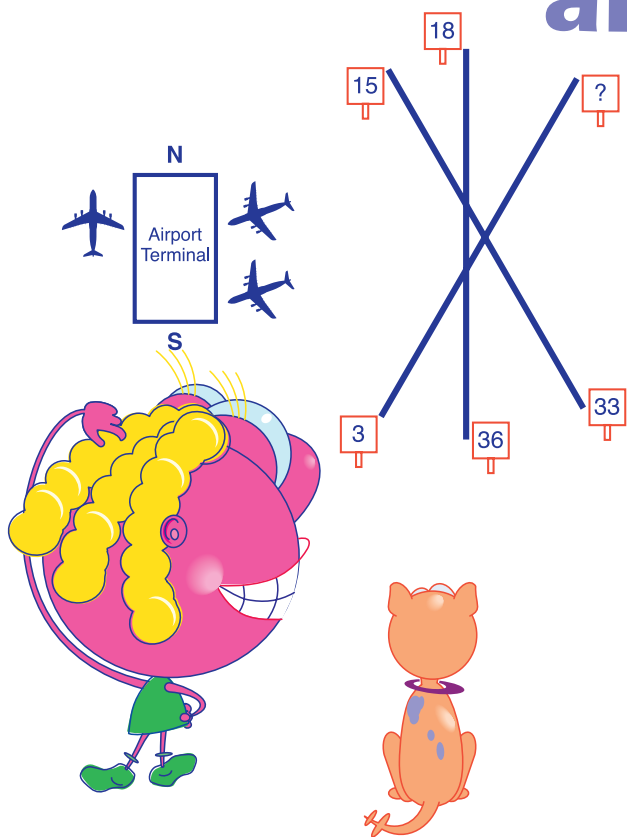


Figure This! Airport runways are labeled with numbers at both ends. The pilot sees the signs when the plane is landing. What is the missing runway number?

Hint: Place a zero after each runway number; then think in terms of degrees. The measure of an angle along a straight line is 180° .

Navigation depends on knowing location and heading. Airplane and ship pilots, surveyors, explorers, and scouts use compasses to find directions.

Figure This!

Get Started:

A heading of 10° means you are traveling in a direction 10° clockwise from due north. The number on the corresponding runway would be 1. Which heading corresponds to a runway labeled 19?

Complete Solution:

- The numbers painted on each end of a runway tell a pilot the compass direction in which to land the plane. For example, consider a runway with ends that face north and south. The north end is labeled with the number 18. Placing a 0 after the 18 to make 180 indicates that a plane landing at this end would be heading south at 180° .
- In the challenge, the runway with the missing number is labeled 3 at its opposite end. The number 3 indicates a heading of 30° . Since each runway is a straight line, the difference between the headings for its two ends is always 180° . So, if one end requires a heading of 30° , the other corresponds to a heading of $30^\circ + 180^\circ$, or 210° . The runway sign, therefore, should be 21.

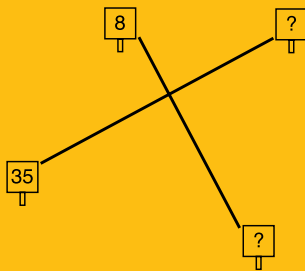
Try This:

- Research websites to learn how the runways are arranged at several different airports. How are the runways numbered at your local airport?
- Identify a building or object that is due north of your home.
- Estimate the compass headings for the street or road nearest your home.

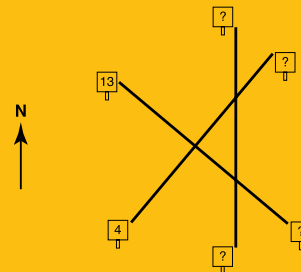
Additional Challenges:

(Answers located in back of booklet)

1. The diagram below shows the runways at a small airport. Use the information given to find the missing runway numbers.



2. Determine the missing runway numbers for the airport shown below.



3. A plane has a 10° heading, turns and goes in the opposite direction. What is the new heading?

Things to Think About:

- Most airports design their runways so that planes are able to take off and land into the wind as much as possible.
- Why are many paved runways made out of concrete and not asphalt?
- How do the surroundings influence where airports are built?
- What length of a runway is required to land a small plane? To land a space shuttle?
- Why do airport runways use 36 instead of 0?

Did You Know That?

- Runways also are labeled with vertical lines, to indicate their length. Each vertical line represents 1000 ft.
- Taxiways—the connections between runways and the terminal building—are usually built at 30° angles to the runways. This allows planes to turn from one to the other without coming to a stop.
- Taxiways are labeled with letters, to indicate the different paths from runways to terminals.
- O'Hare International Airport in Chicago, Illinois, is one of the busiest airports in the world, with more than 900,000 take-offs and landings per year.
- The United States has more than 18,000 airports. More than 8000 have paved runways, while many of the others have grass runways. Approximately 4900 have lighted runways.
- Parallel runways are labeled R and L. For example, two north-south runways might be labeled R18-36L and L18-36R.
- Most interstate highways in the United States are numbered according to the following system: east-west highways are even-numbered, with the numbers increasing from south to north; north-south highways are odd-numbered, increasing from west to east.



Figure This!

Math Challenges for Families

Does Polygon [?] need a jacket?

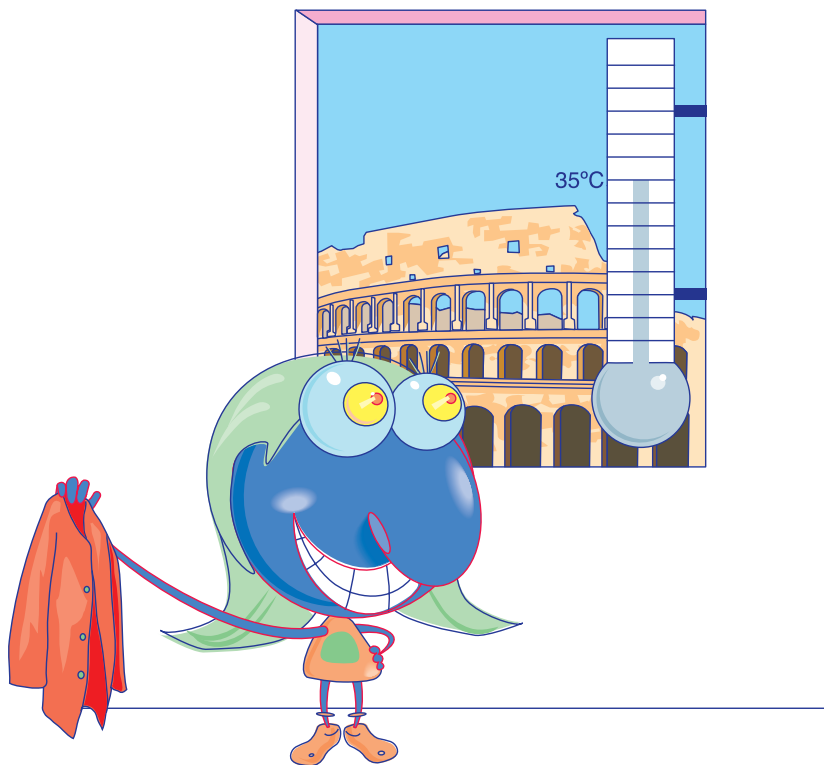


Figure This! Before leaving her hotel, Polygon noticed that the outside temperature was 35° Celsius. Does she need a jacket for the day?

Hint: On the Celsius scale, water freezes at 0° and boils at 100°. On the Fahrenheit scale, water freezes at 32° and boils at 212°.

Having benchmarks in metric units of measurement is important in a global economy. Scientists, doctors, nurses, economists and farmers all need to have a working knowledge of international measuring systems.

Polygon does not need a jacket, since 35° C is 95° F.

Answer:

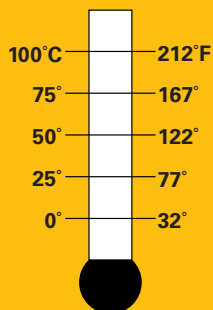
Figure This!

Get Started:

Sketch two thermometers side by side. Label one with the freezing and boiling points in degrees Celsius, the other with the same points in degrees Fahrenheit. Then determine the temperatures halfway between freezing and boiling on each thermometer. A change in degrees Celsius is proportional to the corresponding change in degrees Fahrenheit.

Complete Solution:

- In the diagram on the Celsius thermometer, the halfway mark is 50°, and the quarter marks are 25°, 50°, and 75° since there are 100 divisions between 0° and 100°. On the Fahrenheit thermometer, there are 180 divisions between 32° and 212°. One half of 180 is 90 and 90+32=122, halfway between. Similarly, the quarter marks are 77°, 122°, and 167°. The half and quarter marks on the scales should match. A temperature of 35° C is well above 77° F so Polygon would not need a jacket.



- To determine the exact temperature, use the formula,

$$F = \left(\frac{9}{5}\right)C + 32.$$

When $C = 35^\circ$, the temperature is $95^\circ F$, and Polygon does not need a jacket.

Try This:

- Examine newspapers, magazines, and other media reports to see how the temperature is measured in different cities around the world.
- Look in your cupboards to see how metric labels are used.
- The speedometers in many cars show speed in miles per hour and in kilometers per hour. What would your local speed limits be in kilometers per hour?

Additional Challenges:

(Answers located in back of booklet)

1. Should Polygon wear a jacket if the temperature is $10^\circ C$?
2. A formula for converting between temperatures in Fahrenheit (F) and Celsius (C) is:

$$C = \frac{5}{9} (F - 32)$$

What is the temperature in degrees Celsius when it is $86^\circ F$?

3. Some scientists have observed a connection between the air temperature and the frequency with which a cricket chirps. If you count the number of times a cricket chirps in 1 minute, divide by 4, then add 37, the result is a good estimate of the temperature in degrees Fahrenheit. If a cricket chirps 120 times in 1 minute, how warm is it?
4. At what temperature would a Celsius thermometer and a Fahrenheit thermometer give the same reading?
5. If 50 mph is about 80 km/h, would you be speeding at 35 km/h in a 25 mph zone?

Things to Think About:

- What is a comfortable temperature for bath water?
- Is the body temperature of a fish the same as the temperature of the surrounding water?
- Why do scientists prefer the metric system to the U.S. conventional system?
- Why has the American public resisted the adoption of the metric system?
- Why is the so-called "normal" body temperature of $98.6^\circ F$ reported to the tenth of a degree?

Did You Know That?

- One easy rule for estimating temperature in Fahrenheit is to double the temperature in Celsius and add 30° . Another simple method is to remember this poem by Carole Greenes, Boston University:

30° is hot,

20° is pleasing,

10° is not,

0° is freezing.

- A meter is a "big yard," while a liter is a "big quart."
- A ton is 2000 lb. A metric ton is 1000 kg. The difference is 200 lb or about 91 kg.
- At sunset, the temperature at the moon's equator is about $58^\circ F$. During the night, it drops to about $-261^\circ F$.
- The highest temperature ever recorded in the United States was $134^\circ F$ on July 10, 1913, at Death Valley, California.
- German physicist Daniel Gabriel Fahrenheit (1686–1736) invented the mercury thermometer in 1714. Before then, thermometers contained alcohol.
- Swedish astronomer Anders Celsius (1701–1744) introduced his scale in 1742. The Celsius scale has become part of the metric system of measurement and is used throughout the world.



FigureThis!
Math Challenges for Families

Does drinking soda affect your health??

“Teenage girls’ soda intake is linked to broken bones”

	Fractures	No Fractures
Drank Cola	38	69
Did Not Drink Cola	5	52

Source: USA Today, Thursday, June 15, 2000.

Figure This! For this study, researchers questioned ninth- and tenth-grade girls at a Boston-area high school. Do the data support the headline?

Hint: What percentage of the teenage girls who drank cola also had fractures?

Data organized in tables help people understand how different categories or groups are related. Tables are used by researchers, businesses, media, and the government to display information about study results, profit and loss, news stories, and census data.

Based on the information given, the headline is believable.

Answer:

Figure This!

Getting Started:

If there is a connection between soda intake and broken bones, you might expect that teenagers who drink cola would be more likely to have fractures than teens who do not drink cola. Use the hint to begin your explorations.

	Had a fracture	Did not have fractures
Drank cola	$\frac{38}{38+69} \approx 36\%$	
Did not drink cola		

Complete solution:

One way to analyze this situation is to consider two groups of teens in the study: those who drank soda, and those who did not. From the table, there are $38 + 69$, or 107 students who drank cola, and $5 + 52$, or 57, who did not. Of the 107 students who drank cola, 38 had fractures, or approximately 36%. Of the 57 students who did not drink cola, 5 had fractures, or about 9%. The information is summarized in the table.

	Had a fracture	Did not have fractures
Drank cola	$\frac{38}{38+69} \approx 36\%$	$\frac{69}{38+69} \approx 64\%$
Did not drink cola	$\frac{5}{57} \approx 9\%$	$\frac{52}{57} \approx 91\%$

Based on this information, students who drink cola seem to be more likely to have fractures. Just because an association exists, however, does not mean that drinking cola causes fractures. Other factors may be involved.

Try This:

- Make a small circle with your thumb and index finger. Hold the circle a full arm's length away. Look through this circle and focus both eyes on an object some distance away. Shut one eye at a time. The eye through which the object remains in the circle is your dominant eye. Survey 30 people about both their dominant eye and their dominant hand (right or left). Record your results in a table. Use the results to describe the association between eye and hand dominance.
- Look for reports of medical studies in the media. If possible, display the data given in a table. Do the data seem to support the claims made in the report?

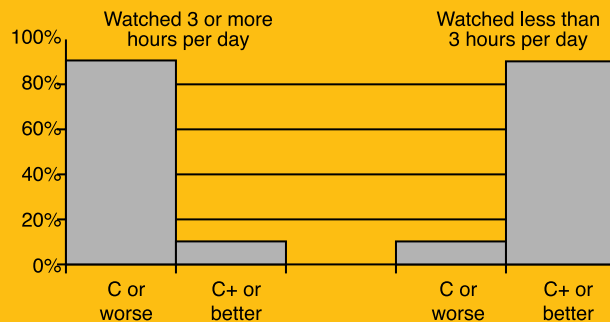
Additional Challenges:

(Answers located in back of booklet)

1. Which of the following bar graphs supports the argument that students who watch less television earn better grades?

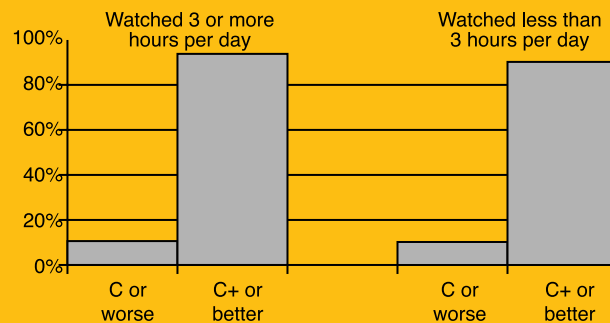
a.

Television and Grades



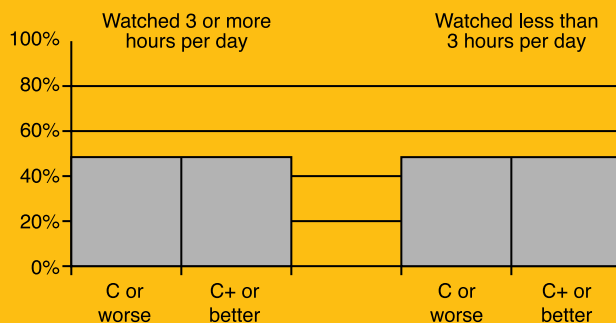
b.

Television and Grades



c.

Television and Grades

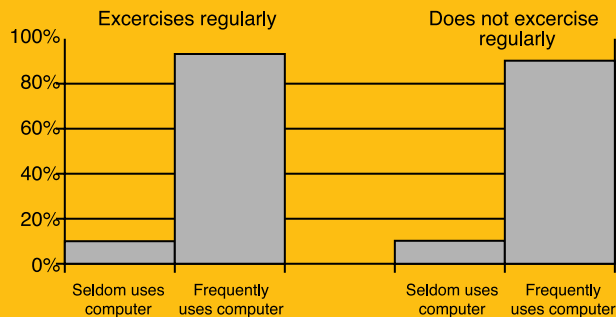


2. In 1999, the two top-earning movies were “Titanic” and “Star Wars, Part IV.” Helix surveyed 200 people who had seen both movies. He recorded the data in a table.

	Liked “Titanic”	Disliked “Titanic”	Totals
Liked “Star Wars”	70		
Disliked “Star Wars”	50	37	
Totals			200

- a. Complete the table above.
- b. Based on this information, do you think that someone who liked “Titanic” would also like “Star Wars, Part IV”?
3. This graph represents survey results about the relationship between the amount of time teen-agers exercise and the amount of time they spend on a computer.

Computer Use and Exercise



What does the graph tell you about the relationship between exercise and computer use?

Things to Think About:

- What factors other than soft-drink intake might affect the frequency of bone fractures in teens?
- Is it easier for you to understand and work with numbers when they are embedded in a paragraph or displayed in a table?
- If there is a strong association between two events, such as hand and eye dominance, does it mean that one causes the other?
- Tables of data as the one given in question 2 of the additional challenges are sometimes called “two-way tables.” For how many cells must you know the entries before you can calculate the remaining values?

Did You Know That?

- Association between events can be positive or negative. Negative association shows that the more of one event, the less of the other, while positive association shows the more of one event, the more of the other.
- To determine if two events are associated, statisticians sometimes use a mathematical technique known as a chi-square test.
- Another tool for organizing information or numbers is a matrix. Matrices look like rectangular tables but typically do not use lines to frame each cell.
- Information that places something in a given category—such as eye color—is called categorical data. Information that can be ordered on a number line—such as height—is called quantitative data.

References:

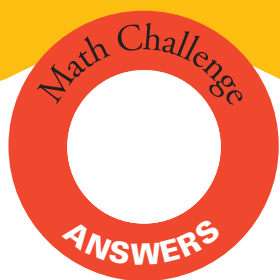
Book:

Hopfensperger, P., Kranendonk, H., and R. Scheaffer. *Probability Through Data*. Dale Seymour Publications, 1999.

Websites:

- www.amstat.org/publications/jse/v2n2/datasets.rossman.html
- www.lib.msu.edu/behm/readmedlit/hypothesis.htm

Notes:



FigureThis!
Math Challenges for Families

Looking for answers?

Here are the answers for the
Additional Challenges section
of each Challenge.

Figure This!

Challenge 65

1. The answer may vary. The pattern shows that the factors to go from one complexion type to another are 3, 2.5, and 2. With this pattern, the next factor is 1.5 giving 45 as the recommended SPF.
2. The potato will be done in about 24 more minutes.
3. About 17 minutes.

Challenge 66

1. The ends of one runway should be labeled 26 and 8; the others should be 35 and 17.
2. The ends of one runway should be labeled 13 and 31; the ends of the other should be 4 and 22. The ends of the north-south runway are 18 and 36.
3. 190° .

Challenge 67

1. Probably, since 10°C is 50°F .
2. 30°C .
3. About 67°F .
4. -40° .
5. No.

Challenge 68

1. Graph a.

2a.

	Liked <i>Titanic</i>	Disliked <i>Titanic</i>	Totals
Liked <i>Star Wars</i>	70	43	113
Disliked <i>Star Wars</i>	50	37	87
Totals	120	80	200

- 2b. For the people in this study, there does not seem to be a strong association between liking one movie and liking the other. About 58%—a little more than half ($70/120$)—of those who liked *Titanic* also liked *Star Wars, Part IV*.
3. The graph reveals little connection between exercise and computer use. Most people in the study frequently use computers, whether they exercise or not.

Challenge 69

1. $1/2$.
2. 0.
3. 100% or 1.
4. less than.
5. $4/6$ or $2/3$

Challenge 70

- 1a. Toyota Prius; it looks like city mpg and highway mpg were reversed.
- 1b. Greatest change is either the Chevrolet Impala or the Saturn SW at 11 mpg, least is Toyota Tacoma at 3 mpg.
- 2a. The equation predicts a highway mpg of about 18.89 or about 19.
- 2b. The prediction is not far off from the actual mileage.
3. Answers may vary, depending on the relative value assigned to field-goal percentage and rebounds. One approach identifies the highest displayed value in each category and uses it as an “ideal.” In this case, the ideal point would be (0.574, 1157). If players are ranked according to this ideal, they fall in the following order: O’Neal, Mutombo, Mourning, Robinson, Divac, Davis, and Ratliff.

Challenge 71

1. The parts are symmetric because the sum of the differences between two sites is the same whether you compare A to B or B to A.
2. The population density of Minnesota is about 60 people per square mile, or about 23 people per square kilometer. This means that the population density in the study area was about 9 times as great.
3. The prices at stores A and E are the most similar, as their sum of differences is only \$0.64. The prices at stores C and E are the most different, with a sum of \$2.38.

Challenge 72

- 1a. 64%
- 1b. 9.8 million
2. About 9.42 million adults ages 18 – 49 watched the show. Of the target group, 72.3% were watching television.
3. The CPM for *ER* was \$38.62; the CPM for *Diagnosis: Murder* was \$792.